Να υπολογιστεί το ολοκλήρωμα

$$\int \frac{dx}{(1-x^2)\sqrt{1-x^2}}$$

ΛΥΣΗ

Θέτουμε $\sqrt{1-x^2} = xt \Rightarrow t = \dfrac{\sqrt{1-x^2}}{x}$

Τότε, $1-x^2 = x^2 \cdot t^2 \Rightarrow 1 = x^2 \cdot t^2 + x^2 \Rightarrow x^2 \cdot (1+t^2) = 1 \Rightarrow$

$$\Rightarrow x^2 = \frac{1}{1+t^2} \Rightarrow 2x\,dx = -\frac{2t}{(1+t^2)^2}\,dt \Rightarrow x\,dx = -\frac{t}{(t^2+1)^2}\,dt$$

$$\Rightarrow \boxed{dx = -\frac{t}{x(1+t^2)}\,dt}$$

Αντικαθιστώντας, παίρνουμε:

$$\int \frac{-\frac{t}{x(1+t^2)}}{x^2 \cdot t^2 \cdot x \cdot t}\,dt = -\int \frac{t}{x^4 \cdot t^3 \cdot (1+t^2)}\,dt =$$

$$= -\int \frac{1}{x^4 \cdot t^2 \cdot (1+t^2)}\,dt = -\int \frac{1}{\frac{1}{(1+t^2)^2} \cdot t^2 \cdot (1+t^2)}\,dt =$$

$$= -\int \frac{1+t^2}{t^2}\,dt = \frac{1}{t} - t + c = \frac{x}{\sqrt{1-x^2}} - \frac{\sqrt{1-x^2}}{x} + c.$$

β' τρόπος

$$\int \frac{dx}{(1-x^2)\sqrt{1-x^2}} = \int \frac{dx}{(1-x^2)(1-x^2)^{1/2}} = \int \frac{dx}{(1-x^2)^{3/2}} \overset{x=\sin u}{=} \int \frac{du}{\cos^2 u} =$$

$$= \tan(\mathrm{Arcsin}\, x) + c.$$